Pattern Recognition Letters 68 (2015) 9–14

Contents lists available at ScienceDirect

Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

Multiple representations and sparse representation for image classification[☆]

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ARTICLE INFO

Article history:
Received 13 February 2015
Available online 18 August 2015

Keywords:
Image classification
Image representation
Sparse representation

ABSTRACT

To extract salient features from images is significant for image classification. Deformable objects suffer from the problem that a number of pixels may have varying intensities. In other words, pixels at the same positions of training samples and test samples of an object usually have different intensities, which makes it difficult to obtain salient features of images of deformable objects. In this paper, we propose a novel method to address this issue. Our method first produces new representation of original images that can enhance pixels with moderate intensities of the original images and reduces the importance of other pixels. The new representation and original image of the object are complementary in representing the object, so the integration of them is able to improve the accuracy of image classification. The image classification experiments show that the simultaneous use of the proposed novel representations and original images can obtain a much higher accuracy than the use of only the original images. In particular, the incorporation of sparse representation with the proposed method can bring surprising improvement in accuracy. The maximum improvement in the accuracy may be greater than 8%. Moreover, The proposed non-parameter weighted fusion procedure is also attractive. The code of the proposed method is available at http://www.yongxu.org/lunwen.html.

1. Introduction

Image representation is an important branch of computer vision. Proper descriptions or representations of images are the basis of achieving good image classification results [1,2]. Once images are well represented, an object in the form of the image can be easily distinguished from the others.

The combination of multiple descriptions or representations of images is effective for improving the classification accuracy [3]. How to obtain competent and complementary multiple descriptions of images is an important topic. Face recognition is a convenient biometric technology and has been widely studied. However, face recognition is still faced with the following challenge. In real-world applications every face may have severe variation of poses, illuminations and facial expressions. As a consequence, images of the same face may have great differences, which usually causes negative effects on face recognition. To get more representations of a face also seems to be a feasible way to improve the accuracy of face recognition and some methods have been proposed for this goal in recent years [4–7]. For instance, the use of symmetrical face images generated from original face images is very useful to overcome the problem of varying appearances of faces [8,9]. The simultaneous use of original face images and their mirror face images can improve the accuracy of face recognition [10,11]. The mirror faces images also seem to be natural faces images and have good visual effects. It appears that a couple of other available representations are also beneficial to face recognition and even noisy faces images are also useful representations of faces [12,13]. The fuzzy logic is proved to be very effective for representations and recognition of face images [14,15]. The fuzzy logic is also competent in other fields such as image processing [16]. The recently proposed modular neural networks are also good tools to represent and recognize faces [17].

Recently proposed sparse representation classification (SRC) algorithms have obtained satisfactory performance in image classification and image super-resolution etc. [18–24]. For comprehensive introductions to sparse representation, please refer to [25–28]. In our opinion, the good performance of SRC algorithms is mainly attributed to the fact that they can do well in determining the intrinsic similarity of objects embedded in high-dimensional image data

[☆] This paper has been recommended for acceptance by Jie Zou.

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http://dx.doi.org/10.1016/j.patrec.2015.07.032
0167-8655/

[8]. In this paper, we call a SRC algorithm conventional SRC algorithm or generalized SRC algorithm. Conventional SRC algorithms are referred to as the l_1, l_0 or $l_p(p < 1)$ minimization based SRC algorithms, in which there is a constraint that the l_1, l_0 or l_p norm of the solution should be minimized. If an algorithm almost has all properties of conventional SRC algorithms but the above constraint is replaced by the l_2 norm of the solution should be minimized or there is no constraint on the solution, we call it a generalized SRC algorithm. A generalized SRC algorithm usually has a closed-form solution but conventional SRC algorithms do not have. Moreover, a generalized SRC algorithm is usually computationally efficient than the conventional SRC algorithm. However, the conventional SRC algorithm usually has a “sparser” solution than the generalized SRC algorithm. Typical conventional sparse representation algorithms include l1-regularized least squares (L1LS) [29], fast iterative shrinkage and thresholding algorithm (FISTA) [30], augmented Lagrangian [31], orthogonal matching pursuit (OMP) [32] etc. Typical generalized sparse representation algorithms include linear regression classification (LRC) [33], collaborative representation (CRC) [34], two phase sparse representation etc. [35–37].

In this paper, we propose a kind of novel representation of images which enhances the importance of pixels with moderate intensities. We for the first time demonstrate that pixels with moderate intensities are very important to distinguish an object from the others. Conventional and generalized sparse representation algorithms are applied with the original images and they proposed novel representations of the images as the image classification algorithms. The experiments especially face recognition experiments show that the proposed novel representations of images are complementary to the original images in representing the object and the combination of these two kinds of representations can lead to a very satisfactory accuracy for image classification and face recognition. Our work has the following merits. (1) It proposes a kind of novel representations of images that is very effective in classifying the images. (2) It devises a completely automatic method to integrate the original images and proposed novel representations. (3) It can achieve a surprising improvement in classification accuracy. This also means that it is a good way to incorporate the proposed novel representations of images with conventional or generalized sparse representation algorithms for image classification.

The remainder of this paper is organized as follows. Section 2 presents the proposed novel representations of images. Section 3 describes the underlying rationale and advantages of the proposed novel representations of images. Section 4 shows the results of extensive experiments. Section 5 provides the conclusions of the paper.

2. The proposed algorithm

2.1. To obtain novel representations of images

The novel representation of an original image is obtained as follows. Let I stand for an original image. Let I_{ij} denote the intensity of the pixel at the i-th row and j-th column of I. Suppose that m is the maximum intensity of all pixels. For a conventional gray image we have $m = 255$. The novel representation of image I is denoted by J and defined as

$$J_{ij} = I_{ij} \cdot (m - I_{ij}) \tag{1}$$

where J_{ij} stands for the intensity of the pixel at the i-th row and j-th column of J. From the definition, we have the following propositions.

Proposition 1. *If I_{ij} is m or zero, then J_{ij} will be zero.*

Proposition 2. *If I_{ij} is an even number, then J_{ij} will have its maximum value when I_{ij} equals to $\frac{m}{2}$.*

It is very easy to prove the above propositions. We can also know that the closer to $\frac{m}{2}$ the I_{ij}, the larger the J_{ij}. As a result, we can conclude that only if a pixel in the original face image is in the range of mid-level intensity, it will be enhanced in the novel representation of the original image; otherwise, it will have a relative small value in the novel representation. Hereafter we also refer to novel representations of original images as virtual images.

2.2. The algorithm to fuse original and virtual images

We describe our algorithm to fuse original and virtual images as follows. After virtual images are obtained, a classification algorithm can be applied to both the original and virtual images, respectively. We use the following flexible score fusion scheme to integrate the classification results. Let $d_o^j(j = 1, \ldots, C)$ denote the distance or dissimilarity (also referred to as score) between the test sample and original face images of the j-th subject. C is the number of all subjects. Let $d_v^j(j = 1, \ldots, C)$ denote the distance (i.e. score) between the test sample and virtual face images of the j-th subject. Let $P_o^1, \ldots, P_o^C$ stand for the sorted results of $d_o^1, \ldots, d_o^C$ and suppose that $P_o^1 \le \cdots \le P_o^C$. Let $P_v^1, \ldots, P_v^C$ stand for the sorted results of $d_v^1, \ldots, d_v^C$ and suppose that $P_v^1 \le \cdots \le P_v^C$. We define $w_{10} = P_o^2 - P_o^1$ and $w_{20} = P_v^2 - P_v^1$. We respectively use $w_1 = \frac{w_{10}}{w_{10}+w_{20}}$ and $w_2 = \frac{w_{20}}{w_{10}+w_{20}}$ as weights of d_o^j and d_v^j. The formula to fuse d_o^j and d_v^j is

$$q_j = w_1 d_o^j + w_2 d_v^j, \quad j = 1, \ldots, C \tag{2}$$

Furthermore, we define

$$r = \arg\min_j q_j \tag{3}$$

Finally the test sample is assigned to the r-th subject. The main steps of our algorithm are presented as follows.

Step 1. Separate all original images into two sets, i.e. the set of training samples and set of test samples.
Step 2. Obtain virtual images of all original images using (1). Then all images are converted into unit column vectors with norm of 1.
Step 3. A classification algorithm is applied to both the original and virtual face images to obtain d_o^j and d_v^j $(j = 1, \ldots, C)$.
Step 4. Obtain weights $w_1 = \frac{w_{10}}{w_{10}+w_{20}}$ and $w_2 = \frac{w_{20}}{w_{10}+w_{20}}$. Integrate d_o^j and d_v^j $(j = 1, \ldots, C)$ using (2).
Step 5. Use (3) to classify the test sample.

2.3. The analysis of the proposed algorithm

Previous study also suggests that we may exploit only a subset of all image pixels for image classification [39]. This somewhat implies that different pixels play different roles in image classification. Furthermore, it also seems that to set different weights to different pixels is reasonable. It should be pointed out that our proposed algorithm indeed also has the idea that different pixels are of different importance in representing the object. We present it in detail below.

From the algorithm description presented in Sections 2.1 and 2.2, we know that the virtual image obtained using our algorithm is very different from the original image. In particular, we know that if the pixel intensity of a region in the original image is very large or small, then the pixel intensity of the same region in the virtual image will be very small. On the other hand, if the pixel intensity of a region in the original image is very close to one-second of the maximum intensity, then the pixel intensity of the same region in the virtual image will be quite close to the maximum intensity. In other words, more emphases will be taken on the pixels with moderate intensities. For a deformable original image such as the face image, the pixel with mid-level intensity may be more stable, so the proposed algorithm is reasonable. Moreover, in order to fully exploit complementary information contained in the original and virtual images, we simultaneously use them to perform image classification. The experiments

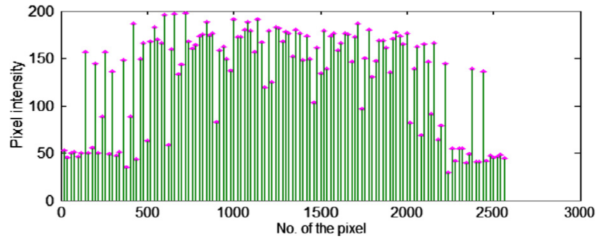

Fig. 1. Original pixel intensities of the first sample of the first subject in the ORL face dataset.

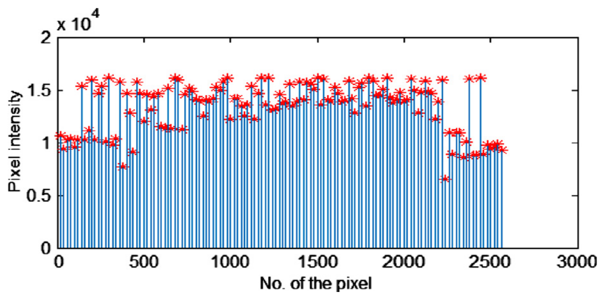

Fig. 2. Pixel intensities of the novel representation of the first sample in the ORL face dataset.

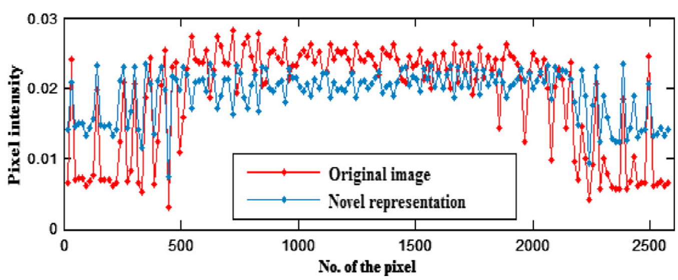

Fig. 3. Normalized original image of the first sample in the ORL face dataset and its normalized novel representation.

presented in Section 4 show that the sparse representation algorithm is very suitable to integrate the original and virtual images to perform image classification.

3. Insight into the proposed algorithm

In this section, we give an intuitive explanation to the rationale of the proposed algorithm. The ORL face dataset was first used to conduct an experiment to intuitively show the difference between the original image and its novel representation proposed in this paper. Fig. 1 shows the original pixel intensities of the first sample of the first subject in the ORL face dataset. Fig. 2 shows the pixel intensities of the novel representation of the same sample. From these two figures we see that the pixels with moderate intensities in the first sample are converted into pixels with very high intensities in the novel representation. Fig. 3 shows normalized original image of this sample in the ORL face dataset and its normalized novel representation. Here “normalized” means that the image vector is converted into unit vectors with the norm of 1. Fig. 3 intuitively illustrates again that the correlation between the original image and its novel representation is not very high.

Fig. 4 shows eight original face images and the corresponding virtual images of a subject in the Georgia Tech face database. We also see that the virtual image is directly associated with the corresponding original face image but they also have clear difference in image appearance. As the original face image and virtual image provide multiple representations with the same face, the simultaneous use of them allows the face to be better described and recognized.

Fig. 4. Eight original face images (first row) and the corresponding novel representations (second row) of a subject in the Georgia Tech face database. For each column, the upper image is an original face image and the lower image is the corresponding novel representation.

The novel representation of an original face image obtained using the proposed algorithm also appears to be a natural virtual face image. Compared with other algorithms to generate virtual face images, our algorithm is very simple and computationally quite efficient. Moreover, there is no any constraint or parameter. However, most of the other algorithms to generate virtual face images are implemented with special constraints or parameters. For instance, the illumination compensation algorithm is established on the basis of a strict assumption and special parameters are needed [38].

4. Experiments and results

We conduct image classification and face recognition experiments to test our method. As shown later, all these experiments demonstrate the feasibility and good performance of our method. Four datasets including a non-face image dataset (i.e. the COIL100 dataset), visual face image and near infrared face image datasets were used. In the experiments, besides collaborative representation, L1LS, FISTA and PALM are directly applied to the original images to perform classification, these algorithms are also applied to the novel representation of the original image for classification. In other words, collaborative representation, L1LS, FISTA and PALM are respectively used as the classification algorithm in Step 3 of the proposed method. In particular, the procedure is as follows: first, the novel representation of each original image is obtained. Then a conventional or generalized SRC algorithm is applied to the novel representation and original image, respectively. Finally, the scores obtained using the SRC algorithms including L1LS, FISTA and PALM and collaborative representation are fused by the algorithm presented in Section 2.2. When collaborative representation is used in our proposed method, we refer to it as “the proposed method with collaborative representation” in the corresponding tables (see Tables 1–4). When collaborative representation, L1LS, FISTA and PALM are directly applied to only the original images, we refer to it as naive collaborative representation, naive L1LS, naive FISTA and naive PALM respectively. We also experimentally compare the proposed new representation with Gabor feature to show the advantage of the new representation (also referred to as our feature). The corresponding comparison experiment is implemented by just first replacing the new representation in our method by the Gabor feature [41], and by then running the other procedures of our method. The same four classification algorithms i.e. collaborative representation, L1LS, FISTA and PALM are also used as classifiers. One can exploit the Gabor wavelet with a certain scale and orientation to perform feature extraction. In our experiments, we select an optimal Gabor filter whose wavelength is 5 and orientation is $\pi/2$ to present Gabor feature.

4.1. Experiment on the COIL100 dataset

In this section, we use the COIL100 dataset to test the proposed method. This dataset contains 7200 images taken from 100 classes and each class has 72 images. Images were taken from several angels. Each image has a resolution of 128 $\times$ 128 pixels. They are all converted into gray images in advance. Fig. 5 shows image examples of a class in the COIL100 dataset.

Table 1
Rate of classification errors (%) on the COIL100 dataset.

Number of training samples per class	1	2	3	4
The proposed method with collaborative representation	51.92	52.56	52.61	52.38
Collaborative representation + Gabor	55.83	56.19	55.80	55.79
Naive collaborative representation	55.37	55.74	55.48	55.24
The proposed method with L1LS	54.66	53.96	53.86	53.57
L1LS + Gabor	57.96	57.24	56.51	55.25
Naive L1LS	58.00	57.00	56.42	56.25
The proposed method with FISTA	52.00	53.56	54.91	55.65
FISTA + Gabor	55.04	59.79	63.06	64.32
Naive FISTA	53.49	54.26	55.35	55.88
The proposed method with PALM	54.86	54.34	54.17	53.84
PALM + Gabor	58.13	57.39	56.67	56.93
Naive PALM	58.04	57.13	56.51	56.60

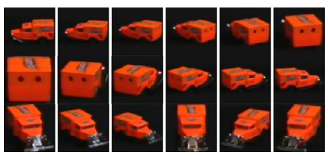

Fig. 5. Image examples of a class in the COIL20 dataset.

Fig. 6. Image examples of two subjects in the ORL dataset.

The first 1, 2, 3 and 4 images of each subject were used as training samples and the others were treated as test samples, respectively. Table 1 shows rates of classification errors of different methods on the COIL100. We see that when collaborative representations, L1LS, FISTA and PALM are integrated with our proposed method, their rates of classification errors are all reduced. This means that the proposed method is very useful for representing the images, and our feature is helpful for improving the accuracy of image classification.

4.2. Experiments on the ORL dataset

In this section the ORL face dataset was used to conduct experiments. This dataset includes 400 face images taken from 40 subjects and each subject has 10 face images. Images of some subjects were taken at different time and have varying lighting, facial expressions (open/closed eyes, smiling/not smiling), and facial details (glasses/no glasses). All images were taken against a dark homogeneous background with the subjects in an upright, frontal position (with tolerance for some side movement). Each image was resized to a 56 by 46 image matrix by using the down-sampling algorithm. Fig. 6 shows image examples of two subjects in the ORL dataset.

The first 1, 2, 3, 4 and 5 face images of each subject were used as training samples and the others were exploited as test samples, respectively. Table 2 shows rates of classification errors on the ORL dataset. This table shows that when the proposed method is integrated with collaborative representation, L1LS, FISTA, PALM, lower rates of classification errors can always be obtained. For example, when naive collaborative representation obtains rates of classification errors of 31.94%, 16.56% and 13.93% for 1, 2 and 3 training samples per class, the integration of the proposed method with collaborative representation respectively obtains rates of classification errors of 23.06%, 12.50% and 11.43% under the same conditions. This demonstrates that the use of the proposed novel representations of original images enables the classification accuracy to be greatly improved. We also easily find our feature obtains lower rate of classification errors than the Gabor feature when four classification algorithms are used as classifies. This means that our feature obtains good performance in face recognition.

4.3. Experiment on the GT dataset

In this section, we use the Georgia Tech face dataset to test the proposed method. The GT face database includes face images of 50 people. All people in the database are represented by 15 color JPEG images with clutter background taken at the resolution of 640×480 pixels. The pictures show frontal and tilted faces with different facial expressions, lighting conditions and scales. Each image was manually labeled to determine the position of the face in the image. Fig. 7 shows image examples in the GT dataset. We use the face images with the background removed and each of these face images has resolution

Table 2
Rate of classification errors (%) on the ORL dataset.

Number of training samples per class	1	2	3	4	5
The proposed method with collaborative representation	23.06	12.50	11.43	8.75	8.50
Collaborative representation + Gabor	33.61	19.38	17.86	12.92	15.00
Naive collaborative representation	31.94	16.56	13.93	10.83	11.50
The proposed method with L1LS	25.28	15.31	14.64	11.67	12.50
L1LS + Gabor	32.78	20.00	19.29	13.75	18.50
Naive L1LS	33.33	19.69	18.93	14.58	13.50
The proposed method with FISTA	25.28	18.75	16.07	10.42	10.50
FISTA + Gabor	34.17	24.06	29.29	28.75	30.00
Naive FISTA	31.67	18.44	16.79	12.08	13.50
The proposed method with PALM	26.11	15.63	14.29	12.50	12.00
PALM + Gabor	33.33	19.69	18.57	14.58	19.50
Naive PALM	33.33	19.69	18.57	13.75	13.00

We also see that our feature is superior to the Gabor feature.

Fig. 7. Image examples in the GT dataset.

Table 3
Rate of classification errors (%) on the GT dataset.

Number of training samples per class	1	2	3
The proposed method with collaborative representation	63.86	52.15	52.17
Collaborative representation + Gabor	74.29	64.92	65.33
Naive collaborative representation	66.57	57.54	54.67
The proposed method with L1LS	65.86	55.54	53.67
L1LS + Gabor	73.57	66.15	64.33
Naive L1LS	68.00	61.08	58.50
The proposed method with FISTA	62.59	54.77	52.83
FISTA + Gabor	71.00	68.46	72.50
Naive FISTA	65.14	54.92	53.50
The proposed method with PALM	65.86	56.31	54.00
PALM + Gabor	73.86	66.46	64.33
Naive PALM	68.14	61.38	58.33

Fig. 8. Image examples of a class in the Lab2 face dataset.

of 40 × 30 pixels. They are all converted into gray images in advance. The first 1, 2 and 3 face images of each subject are used as training samples and the other images are test samples. Table 3 shows the experimental results. From this table, we see that the proposed method can decrease the rate of errors. For example, when naive L1LS obtains rates of classification errors of 68.00%, 61.08% and 58.50%, the proposed method with L1LS obtains rates of classification errors of 65.86%, 55.54% and 53.67%, respectively.

4.4. Experiment on the Lab2 dataset

In this section, we use the near infrared face image dataset from the Lab2 dataset, to test the proposed method. This dataset contains 1000 images taken from 50 subjects and each subject has 20 images. Each image has a resolution of 200 × 200 pixels. The description of the dataset is available in [40] and the Web page: http://www.yongxu.org/databases.html. Fig. 8 shows image examples of a face in the Lab2 face dataset. The first 1, 2, 3 and 4 images of each subject were used as training samples and the others were treated as test samples, respectively. Table 4 shows the rates of classification errors of different methods. We see again that the proposed method is very beneficial to the decrease of the rate of classification errors. Moreover, our feature performs better than the Gabor feature.

5. Conclusions

The feasibility and effectiveness of the proposed novel representation of images are demonstrated by extensive image classification experiments including face recognition experiments. The proposed algorithm can be used as a general means to exploit the original representation, in the form of image to obtain alternative representation of objects. The proposed algorithm has wide applicability. The proposed method not only exploits the original representation to easily produce alternative representation, but also is easy to implement and has a low computational cost. Moreover, it allows the accuracy of image classification to be greatly improved and the maximum improvement in the accuracy may be greater than 8%. Another advantage of the proposed algorithm is that both the procedure to generate the novel representation of the original image and the procedure to fuse the novel representation and original image for classification have no parameter. Thus, they are mathematically very tractable and the proposed algorithm is a completely automatic algorithm without any manual setting.

Table 4
Rate of classification errors (%) on the Lab2 near dataset.

Number of training samples per class	1	2	3	4
The proposed method with collaborative representation	40.21	28.22	27.65	24.25
Collaborative representation + Gabor	52.84	33.56	33.18	28.38
Naive collaborative representation	41.68	31.00	29.76	26.00
The proposed method with L1LS	41.79	31.56	27.41	24.12
L1LS + Gabor	46.42	33.44	31.18	25.50
Naive L1LS	42.74	33.11	31.65	25.50
The proposed method with FISTA	42.32	35.67	35.18	32.75
FISTA + Gabor	46.74	43.44	47.41	45.62
Naive FISTA	44.11	36.78	36.24	36.38
The proposed method with PALM	42.42	30.56	29.41	24.50
PALM + Gabor	47.79	33.78	32.12	28.75
Naive PALM	44.11	33.11	32.00	27.63

Acknowledgments

The work reported in this paper is partly supported by Science and Technology Development Fund (FDCT) of Macao SAR (FDCT/128/2013/A), NSFC under Grant nos. 61370163, 61233011, 61300032 and 61332011, as well as the Shenzhen Municipal Science and Technology Innovation Council under Grant nos. JCYJ20130329151843309, CXZZ20140904154910-774, and JCYJ20140904154645958.

References

[1] J. Chen, S.-G. Shan, C. He, G.-Y. Zhao, M. Pietikäinen, X.-L. Chen, W. Gao, WLD: a robust local image descriptor, IEEE Trans. Pattern Anal. Machine Intell. 32 (9) (2010) 1705–1720.
[2] X.-P. Hong, G.-Y. Zhao, M. Pietikäinen, X.-L. Chen, Combining LBP difference and feature correlation for texture description, IEEE Trans. Image Process. 23 (6) (2014) 2557–2568.
[3] Alex F. Mansano, J.A. Matsuoka, Luis C.S. Afonso, João P. Papa, Fábio Augusto Faria, Ricardo da Silva Torres, Improving image classification through descriptor combination, in: Proceedings of SIBGRAPI, 2012, pp. 324–329.
[4] W. Gao, S.-G. Shan, X.-J. Chai, X.-W. Fu, Virtual face image generat-ion for illumination and pose insensitive face recognition, in: Proceedings of International Conference on Acoustics, Speech and Signal Processing, ICASSP, 2003.
[5] B. Tang, S. Luo, H. Huang, High performance face recognition system by creating virtual sample, in: Proceedings of International Conference on Neural Networks and Signal Processing, 2003, pp. 972–975.
[6] Y.-S. Ryu, S.-Y. Oh, Simple hybrid classifier for face recognition with adaptively generated virtual data, Pattern Recognit. Lett. 23 (7) (2002) 833–841.
[7] N.P.H. Thian, S. Marcel, S. Bengio, Improving face authentication using virtual samples, in: Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing, 2003, pp. 6–10.
[8] Y. Xu, X. Zhu, Z. Li, G. Liu, Y. Lu, H. Liu, Using the original and 'symmetrical face' training samples to perform representation based two-step face recognition, Pattern Recognit. 46 (2014) 1151–1158.
[9] S. Wu, J. Cao, 'Symmetrical face' based improved LPP method for face recognition, Optik 125 (2014) 3530–3533.
[10] Y. Xu, X. Li, J. Yang, D. Zhang, Integrate the original face image and its mirror image for face recognition, Neurocomputing 131 (5) (2014) 191–199.
[11] Y. Xu, X. Li, J. Yang, Z. Lai, D. Zhang, Integrating conventional and inverse representation for face recognition, IEEE Trans. Cybern. 44 (10) (2014) 1738–1746.
[12] Y.-Y. Wang, M. Wang, Y. Chen, Q. Zhu, A novel virtual samples-based sparse representation method for face recognition, Optik 125 (2014) 3908–3912.
[13] C. Sanderson, K.K. Paliwal, Noise compensation in a person verification system using face and multiple speech feature, Pattern Recognit. 36 (2) (2003) 293–302.

[14] P. Melin, O. Castillo, A review on type-2 fuzzy logic applications in clustering, classification and pattern recognition, Appl. Soft Comput. 21 (2014) 568–577.
[15] P. Melin, O. Mendoza, O. Castillo, Face recognition with an improved interval type-2 fuzzy logic sugeno integral and modular neural networks, IEEE Trans. Syst. Man Cybern. Part A 41 (5) (2011) 1001–1012.
[16] P. Melin, C.I. González, J.R. Castro, O. Mendoza, O. Castillo, Edge-detection method for image processing based on generalized type-2 fuzzy logic, IEEE Trans. Fuzzy Syst. 22 (6) (2014) 1515–1525.
[17] G.E. Martinez, P. Melin, O.D. Mendoza, O. Castillo, Face recognition with choquet integral in modular neural networks, Recent Adv. Hybrid Approaches Des. Intell. Syst. 547 (2014) 437–449.
[18] J. Wright, Y. Ma, J. Mairal, G. Sapiro, T.-S. Huang, S.-C. Yan, Sparse representation for computer vision and pattern recognition. Proc. IEEE, 98, 2010, pp. 1031–1044.
[19] J. Wright, A. Yang, A. Ganesh, S.S. Sastry, Y. Ma, Robust face recognition via sparse representation, IEEE Trans. Pattern Anal. Mach. Intell 31 (2009) 210–227.
[20] J. Yang, J. Wright, T. Huang, Y. Ma, Image super-resolution via sparse representation, IEEE Trans. Image Process. 19 (2010) 2861–2873.
[21] J. Yang, J. Wright, T. Huang, Y. Ma, Image super-resolution as sparse representation of raw image patches, in: Proceedings of IEEE Conference on Computer Vision and Pattern Recognition, 2008, pp. 1–8.
[22] B. Han, F. Wu, D. Wu, Image representation by compressive sensing for visual sensor networks, J. Vis. Commun. Image Represent. 21 (4) (2010) 325–333.
[23] Z. Fan, M. Ni, Q. Zhu, C. Sun, L.-P. Kang, L0-norm sparse represent-ation based on modified genetic algorithm for face recognition, J. Vis. Commun. Image Represent. 28 (2015) 15–20.
[24] C.-Y. Lu, H. Min, J. Gui, Face recognition via weighted sparse repress-entation, J. Vis. Commun. Image Represent. 24 (2) (2013) 111–116.
[25] Z. Zhang, Y. Xu, J. Yang, X. Li, D. Zhang, A survey of sparse representation: algorithms and applications, IEEE Access 3 (2015) 490–530.
[26] H. Cheng, Z. Liu, L. Yang, X. Chen, Sparse representation and learning in visual recognition: theory and applications, Signal Process., 93(6) (2013) 1408–1425.
[27] M. Schmidt, G. Fung, R. Rosales, 2009. Optimization methods for l1-regularization, University of British Columbia, Technical Report TR-2009-19.
[28] J.A. Tropp, A.C. Gilbert, M.J. Strauss, Algorithms for simultaneous sparse approximation. Part I: greedy pursuit, Signal Process. 86 (3) (2006) 572–588.
[29] S.P. Boyd. http://web.stanford.edu/~boyd/l1_ls/, 2008 (accessed 04.08).
[30] P.-H. Gong, C.-S. Zhang, Z.-S. Lu, J.-H. Huang, J.-P. Ye, A general iterative shrinkage and thresholding algorithm for non-convex regularized optimization problems, in: Proceedings of International Conference on Machine Learning, ICML, 2013, pp. 37–45.
[31] H.-Z. Luo, H.-X. Wu, G.-T. Chen, On the convergence of augmented Lagrangian methods for nonlinear semi-definite programming, J. Global Optim. 54 (3) (2012) 599–618.
[32] D. Needell, R. Vershynin, Signal recovery from inaccurate and incomplete measurements via regularized orthogonal matching pursuit, IEEE J. Sel. Topics Signal Process. 4 (2) (2010) 310–316.
[33] I. Naseem, R. Togneri, M. Bennamoun, Robust regression for face recognition, Pattern Recognit. 45 (1) (2012) 104–118.
[34] L. Zhang, M. Yang, X. Feng, Sparse representation or collaborative representation: which helps face recognition? in: Proceedings of IEEE International Conference on Computer Vision, 2011, pp. 471–478.
[35] Y. Xu, D. Zhang, J. Yang, J.-Y. Yang, A two-phase test sample sparse representation method for use with face recognition, IEEE Trans. Circuits Syst. Video Technol. 25 (2011) 1255–1262.
[36] F. Dornaika, Y. Traboulsi, C. Hernandez, A. Assoum, Self-optimized two phase test sample sparse representation method for image classification, in: Proceedings of the 2nd International Conference on Advances in Biomedical Engineering, 2013, pp. 163–166.
[37] F. Dornaika, Y. Traboulsi, A. Assoum, Adaptive two phase sparse representation classifier for face recognition, in: Proceedings of Advanced Concepts for Intelligent Vision Systems, 2013, pp. 182–191.
[38] P.-C. Hsieh, P.-C. Tung, Shadow compensation based on facial symmetry and image average for robust face recognition, Neuro-comput. 73 (13) (2010) 2708–2717.
[39] I. Smielik, K.-D. Kuhnert, Statistical dependence of pixel intensities for pattern recognition, in: Proceedings of IEEE International Conference on Industrial Technology, 2013, pp. 1179–1183.
[40] Y. Xu, A. Zhong, J. Yang, D. Zhang, Bimodal biometrics based on a representation and recognition approach, Opt. Eng. 50 (3) (2011) 037202–037202.
[41] P. Koves. http://www.csse.uwa.edu.au/~pk/Research/MatlabFns/, 2006 (accessed 10.06).