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Multiple vector representations of images and robust dictionary learning

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ABSTRACT

In this paper novel multiple vector representations of images are proposed and a robust dictionary learning method is designed. The multiple vector representation scheme enables an image to be observed with multiple views. Moreover, multiple vector representations are directly generated from the original image via a simple and efficient way whereas multi-view data usually have a high acquiring cost. The proposed method applies the same dictionary learning algorithm to the multiple vector representations and designs a very reasonable weighted logarithmic sum scheme to integrate classification scores of all vector representations. Main merits of this work are in the following points. First, it offers a quite novel viewpoint to take insight into representation of objects. It for the first time reveals that rows and columns of images can be viewed as two different sequences and pixel arrangements in terms of rows and columns of the image allow the object to be observed with two different angles of view. Second, it shows that when conventional dictionary learning algorithms are combined with the proposed multiple vector representations and weighted logarithmic sum scheme, very robust and accurate classification results can be obtained. This also partially means that diversity of vector representations of the image can be further consolidated by matrix decomposition in dictionary learning, so the resultant complementary information can be better exploited. Third, differing from conventional research routines, our study tells us that to fully dig and exploit possible representation diversity might be a better way to lead to potential various appearance and high classification accuracy of the image. The code of the proposed method is accessible at http://www.yongxu.org/lunwen.html.

1. Introduction

Dictionary learning mainly aims at obtaining reliable features and brief presentations of samples [5,9,32]. After getting dictionaries and features (also called presentation coefficients) from the original samples, we can exploit them for classification, image processing, retrieval and searing tasks etc[4,21]. Dictionary learning algorithms can be categorized into several kinds, such as supervised and unsupervised dictionary learning [33], discriminative and locality constrained dictionary learning [14,17,40]. For some applications, it is important to make the dictionary have special structure or be a modular dictionary [15,16,39].

Robust dictionary learning is of significance [24,29,30]. Robustness not only means that the obtained dictionary and features are very effective to resist noise and outliers, but also the algorithm can bring satisfactory recognition results for pattern classification tasks. For face recognition, a widely studied image classification problem, typical examples of noise also include variations and difference of face images of the same individual caused by changeable illuminations, facial expressions and poses. To exploit proper constraints seems to be a feasible strategy to overcome noise and outliers [13,18]. Labels related and atoms associated constraints have been used for this purpose [17].

When applied to image data, the dictionary learning algorithm should first transform each image into a vector. Actually, many other methods also need to transform each image into a vector when dealing with images [12,37]. It is obvious that there may be different ways to perform the transform. For example, we can concatenate the first to last rows in sequence of an image matrix to attain a vector. Alternatively, we can concatenate the first to last columns of an image matrix to attain another vector. Moreover, for dictionary learning, because of consequent matrix calculation, even

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if the concatenating result of the last to first rows of an image matrix is different from the concatenating result of the first to last rows.

Most of attentions of researchers are paid on seeking good algorithms including representation algorithms and classification algorithms. On the other hand, under the condition of a fixed algorithm, to best exert and exploit the original data to achieve the best performance is somewhat ignored. For image data, potential multiple representations directly from raw data of course can provide different observations of a sample. When the same dictionary learning algorithm is applied to them, it is quite possible to obtain better results owing to their complementarity. Multiple representations are partially similar with the multiple views [8,10,42], but only little calculation cost needs to be consumed when multiple representations directly from raw data are produced.

In recent years, we have recognized that potential multiple samples of an original sample are very useful for computer vision and classification tasks. Typical examples of potential multiple samples include virtual face samples and synthesized face samples [27,28,31]. They can help to improve the accuracy of face detection and the performance of face recognition. Many virtual and synthesized face samples seem to like true faces, though they can be obtained via simple schemes. For instance, symmetry face images can possess the symmetrical structure of true faces [38]. Of course, elaborated algorithms can also be exploited to produce virtual or synthesized face samples [36]. Both potential multiple samples and multiple representations can offer more appearance presentations of raw data. However, they are different because potential multiple samples mean that alternative samples are generated from an original sample but multiple representations just are arrangement results of entries of an original sample.

As we know, for deformable objects, potential multiple samples are absolutely important, because they provide possible variations of the original sample. Especially, under the condition that only few original samples are available for a deformable object, potential multiple samples will be very helpful for improving the accuracy of classification, searching and retrieval. Compared with multiple samples, multiple representations are easier to produce and no any algorithm and trick are needed.

For dictionary learning on images, when the algorithm operates the data matrix consisting of sample vectors, variety of different arrangement results of an original sample is indeed further increased. In this sense, we can increase variety of data in a twofold way. Firstly, multiple representations generated from the original image sample allow an original sample to have different representations. Secondly, base on the matrix decomposition result in dictionary learning, reconstructions of multiple representations of an image sample might have greater difference than that of the original multiple representations. Thus, dictionary learning further enhances the variety of data, so the simultaneous use of the dictionary learning algorithm and multiple representations is beneficial to correct classification. This point is main motivation of this work.

In this paper, we propose a novel dictionary learning algorithm for images based on potential multiple representations of images. In Section 2, we present the proposed method. In Section 3, we show rationales of the proposed method. Section 4 describes the experimental results. Section 5 offers a brief conclusion.

2. Proposed method

2.1. Main steps of the proposed method

In this section, we first present main steps of the proposed method as follows.

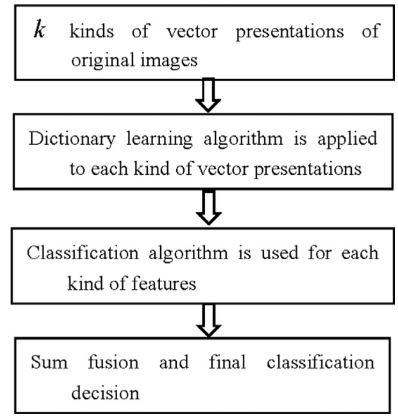

Fig. 1. Flowchart of the proposed method.

Step 1. Multiple representations in the form of vectors are directly generated from original images. In particular, suppose that there are t kinds of vector representations of original images. Each kind of vector representations of original images is generated using a certain transform scheme. Though we just set $t = 4$ in Section 2.2, it has more possible values and there are other ways to obtain multiple representations. For example, converting an original image into a new image pixel by pixel is also a feasible approach to obtain alternative representations [33,35].

Step 2. The same dictionary learning algorithm is applied to each kind of vector representations of original images. In other words, since there are t kinds of vector representations of original images, then we have t kinds of training samples and test samples. As a result, t kinds of features of these training samples and test samples are generated from the same dictionary learning algorithm.

Step 3. The same classification algorithm is respectively used for the t kinds of features of these training samples and test samples. And for each kind of features, classification scores of all test samples with respect to different classes are obtained.

Step 4. For a test sample, the proposed sum fusion scheme is employed to add classification scores of this test sample with respect to the same class. The sum result is referred to as final score of the test sample with respect to this class. Finally, the test sample is classified into the class with the minimum final score.

The flowchart of the proposed method is shown in Fig. 1.

2.2. Details of the proposed method

We provide details of the proposed method in this subsection. First of all, for the $r-th$ kind of samples, we denote the $i-th$ training sample in the form of vector by y_r^i and denote the test sample in the form of vector by p_r. Both y_r^i and p_r are generated from image matrices. Moreover, for the $r-th$ kind, features of y_r^i and p_r obtained using the dictionary learning algorithm are x_r^i and q_r, respectively. The dictionary corresponding to the $r-th$ kind of samples is expressed as D_r.

The following texts interpret Step 1. Let image matrices to produce y_r^i and p_r be Y_r^i and P_r are respectively. y_r^i and p_r produced in four ways. Thus, there are four kinds of samples in total. The first way respectively concatenates the first to last rows in sequence of image matrices Y_r^i and P_r to obtain y_r^i and p_r. The second way respectively concatenates the last to first rows in sequence of the same image matrices to obtain y_r^i and p_r. The third way respectively concatenates the first to last columns in sequence of the same image matrices to obtain y_r^i and p_r. The fourth way respectively concatenate the last to first columns in sequence of the same image matrices to obtain y_r^i and p_r.

In Step 3, the scores are obtained using the classification algorithm in the following way. Let label matrix of the training samples be L. The $j-th$ row, i.e. row vector L_j of L, represents the class label of the $j-th$ training sample. If the $j-th$ training sample belongs to the $k-th$ class, then only the $k-th$ entry of L_j is 1 and all other entries are zeroes. For a kind of samples, features of all the samples form matrix X. Suppose that matrix W can approximately transform X into L, then we have model $XW=L$. We require that W has a minimal norm, so we have a solution of $W=(X^TX+\gamma I)^{-1}X^TL$,where γ is a small positive constant. For test sample q_r, we calculate its desired label via $l_r=q_rW$. Then we denote the score of the test sample with respect to the $g-th$ class by $d_r^g=||l_r-label_g||$. $label_g$ is the class label of the $g-th$ class, in which only the $g-th$ entry is 1 and all other entries are zero.

In Step 4, the formula of the sum fusion scheme is $score_g=\sum_{r=1}^{k} w_r log(d_r^g)$. $log(d_r^g)$ denotes the natural logarithm of d_r^g. If $h=\arg\min_g score_g$, then the test sample is classified to the $h-th$ class. w_r is calculated as follows. We sort d_r^1 ... d_r^C in the ascending order and write down the sorting result as e_r^1 ... e_r^C, then w_r is defined as e_r^1 - e_r^2.

3. Rationale of the proposed method

The proposed method finds a way to fully exploit representation diversity of original data for dictionary learning, whereas almost all other dictionary learning methods just try the best to exert performance of the method itself based on single representation of original data.

Our main motivation is that because the appearance of an object is reflected by data, different data indeed provide different appearances for us. When image matrices are transformed into vectors, different transform ways indeed lead to different vector representations. In other words, one image will produce multiple representations for the object, and these presentations can be understood as different observations of the same object. In this sense, simultaneous use of these multiple representations allows information of the object to be better exploited.

The matrix decomposition implemented by the dictionary learning algorithm further enforces difference of multiple representations of the object. Specifically, since a dictionary learning algorithm decomposes sample matrix P into dictionary matrix D and feature matrix X and the multiplication result of D and X approaches P, the difference in different D and X usually is greater than the difference in different P. Intuitively, $||Y||\approx||DX||\leq||D||\cdot|X||$ partially implies that after matrix decomposition the data may have greater range of norm, which means greater difference in data.

The proposed multiple vector representations are partly similar with multi-view data, as both of them enables different aspects of an individual to be observed. However, multi-view data need higher cost of data collection, but multiple vector representations do not and are directly obtained from original images. From another viewpoint, if we review different rows of an image matrix successively comes in terms of time, then we can concatenate all rows to form a vector representation. Alternatively, if we review different columns of an image matrix successively comes, then we can concatenate all columns to form another vector representation. These two vector representations are not only different, but also have respective physical meaning in representing the image.

The proposed sum fusion scheme is reasonable owing to the following factors. First, to take the result of the second minimum of scores minus the smallest score as the weight is proper. The score in this paper means dissimilarity. Since we exploit the smallest score to determine the classification label of a test sample. the smaller, the smallest score the greater, the probability of the classification decision being correct. Moreover, if the second minimum of scores is much greater than the smallest score, the classification decision also has a high confidence. This point has been partially explained in [34]. An approach was also used for confidence measures in identity varication [2]. Second, the logarithm of the score not only keeps the numerical order of the scores, but also enlarges the difference of small scores [25]. As a result, in the sum fusion, the dominant role of small scores can be kept, which is beneficial to obtain correct classification. For example, if there are two original scores, 0.2 and 0. 1, the difference value is 0.1. However, $log(0.2)$ and $log(0.1)$ are equal to -1.6094 and -2.3026, respectively, so the difference value becomes 0.6932, much greater than the original difference value i.e. 0.1.

Besides the approach to arrange entries of a raw sample to form alternative representations, researchers can also view features learned from the raw data as new representations. For example, the binary descriptions can be used as effective new representations [6,20]. The features can be produced with a specific constraint [6] and also may lead to good classification performance [7,19,20]

A little flaw of the proposed method is that it has a higher computational cost than that of the conventional dictionary learning algorithm. In particular, because there are four kinds of representations, the computational cost of the proposed method is four times that of the conventional dictionary learning algorithm.

4. Experiments

In order to show the performance of our proposed algorithm, we implement the proposed algorithm on the K-SVD [1], D-KSVD [41], LC-KSVD [14] and FDDL [40] algorithm, and denote them as the improvement to K-SVD, improvement to D-KSVD, improvement to LC-KSVD and improvement to FDDL. Moreover, the Extended Yale B face database [11], the PIE face database [26], the AR face database [22] and COIL 20 [23] are used in this experiment. Each algorithm is carried out ten times and the average accuracy is reported. For every algorithm, we obtain its optimal parameter values via the grid search scheme (e.g., Table 1).

Table 1
Average accuracy(%) on the Yale B face database.

Number of atoms	38	76	114	152	190	228	266	304	342	380
KSVD	55.2	68.0	75.1	80.2	82.9	85.3	86.2	83.6	52.6	88.7
Improvement to KSVD	58.1	72.4	80.1	84.0	86.5	87.9	88.3	87.0	63.8	88.9
DSVD	52.4	68.3	76.4	80.9	84.1	85.8	87.4	87.1	85.5	81.3
Improvement to DSVD	54.1	72.9	81.5	85.0	87.2	88.5	89.2	88.9	88.2	87.1
LC-KSVD	55.3	69.3	77.9	81.5	84.0	85.6	86.6	87.9	88.4	87.2
Improvement to LC-KSVD	55.4	72.9	80.5	83.9	85.9	87.0	87.5	87.9	87.9	87.5
FDDL	45.7	70.2	78.5	83.1	85.2	86.3	87.0	87.1	87.4	87.4
Improvement to FDDL	50.0	71.6	80.9	85.7	88.4	89.9	90.6	91.1	91.2	90.7

Table 2
Average accuracy(%) on the PIE face database.

Number of atoms	68	136	204	272	340
KSVD	63.7	68.1	71.4	74.9	77.2
Improvement to KSVD	68.4	75.4	78.6	81.2	82.4
DSVD	63.8	68.3	73.1	75.3	77.7
Improvement to DSVD	69.2	77.7	81.1	82.5	83.4
LC-KSVD	62.8	73.0	72.6	74.3	76.4
Improvement to LC-KSVD	67.9	79.3	77.7	76.5	75.5
FDDL	47.6	69.9	75.5	78.5	80.1
Improvement to FDDL	57.2	76.5	81.4	82.9	83.8

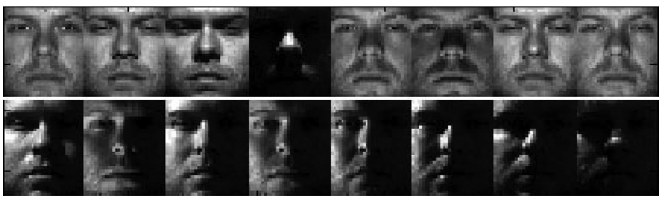

Fig. 2. Some examples images of the Extended Yale B face database.

Fig. 3. Some examples images of the PIE face.

4.1. Experiments on the Extended Yale B face database

The Extended Yale B face database consists of 2,414 front-face images of 38 people and these images were captured under various illumination conditions and expressions. Each person has 59 to 64 images and each image is converted into the size of 32×32 pixels. Some face images from this database are shown in Fig. 2. 10 images of each person are randomly chosen as training samples and the remaining images are exploited for testing. For the K-SVD and D-KSVD algorithms, the number of atoms varies from 38 to 380 with an interval of 38. For the FDDL algorithm, the number of atoms of each class sub-dictionary changes from 1 to 10.

4.2. Experiments on the PIE face database

The PIE face database consists of 41,368 front-face images of 68 persons, and the face images of each person were captured under 13 different poses, 43 different illumination conditions, and 4 different facial expressions. Several sample images from the PIE face database are shown in Fig. 3. Following [3], we choose the five near-frontal poses (C05, C07, C09, C27, C29) of each subject and use all the images under different illumination conditions and facial expressions. Thus we obtain 170 images for each individual. Every image is normalized to the size of 32×32. We randomly select ten images of each person as training samples and use the remaining samples as test samples. The average recognition rates are reported in Table 2.

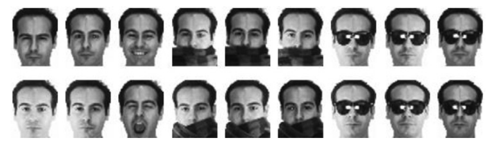

Fig. 4. Some examples images of the AR face.

4.3. Experiments on the AR face database

The AR face database contains more than 4,000 images of 126 persons, which were captured in two sessions. Each person has 26 face images, and each face image is captured under various lighting conditions. Following [33], a subset of the AR face database is used, which consists of 3,120 images from 120 persons (65 men and 55 women). The size of the AR images is 40×50. The images of one person from the AR face database are shown in Fig. 4.

We randomly select 10 face images of each person as training samples and the rest sixteen images of each person are used for testing. The average recognition rates are given in Table 3.

4.4. Experiments on the COIL 20 database

The COIL20 dataset includes different views of 20 objects under different lighting conditions. Following [3], each image is resized to 32×32 pixels and the challenge of this dataset is evaluated on alternative viewpoints. Several sample images from the COIL20 database are shown in Fig. 5.

We randomly select 10 images per object as training samples and the remaining images are treated as test samples. The experiment results are shown in Table 4.

Table 3
Average accuracy(%) on the AR face database.

Number of atoms	120	240	360	480	600	720	840	960	1080	1200
KSVD	85.3	89.9	93.1	94.3	95.2	96.0	96.4	96.1	79.8	97.2
Improvement to KSVD	91.0	94.8	96.4	97.0	97.3	97.6	97.6	97.4	80.1	97.2
DSVD	82.8	89.5	92.0	93.6	94.6	95.4	96.2	96.0	93.1	96.7
Improvement to DSVD	90.2	94.3	95.5	96.3	96.6	97.0	97.3	97.2	96.1	97.1
LC-KSVD	86.8	90.2	92.7	93.7	94.5	94.9	95.4	95.4	95.8	92.4
Improvement to LC-KSVD	88.7	92.3	94.0	94.5	94.7	94.8	94.9	94.9	94.8	93.4
FDDL	83.4	91.4	92.2	93.1	93.5	93.9	93.9	93.9	93.9	93.9
Improvement to FDDL	86.6	94.1	95.4	95.9	96.0	96.2	96.4	96.4	96.5	96.1

Table 4
Average accuracy(%) on the COIL 20 face database.

Number of atoms	40	60	80	100	120	140	160	180	200
KSVD	82.5	85.5	87.2	88.0	88.4	89.0	89.5	84.7	89.0
Improvement to KSVD	84.0	86.8	88.2	89.1	89.3	89.5	89.8	90.3	89.0
DSVD	84.3	85.6	86.9	87.5	88.5	89.1	88.7	87.0	84.1
Improvement to DSVD	84.6	86.8	88.0	89.0	89.6	90.0	90.0	89.9	89.0
LC-KSVD	84.7	87.8	88.3	88.7	89.2	89.1	89.1	88.9	89.4
Improvement to LC-KSVD	85.3	88.9	89.7	90.2	90.5	90.6	90.5	90.8	90.5
FDDL	84.5	87.8	88.2	89.2	89.9	89.9	90.2	90.6	91.1
Improvement to FDDL	80.5	86.5	89.1	89.1	91.1	91.3	91.7	91.9	91.8

Fig. 5. Some example images of the COIL20 database.

5. Conclusions

In this paper, we propose multi-view-like multiple vector representations for images and design a robust dictionary learning method on basis of them. The multiple vector representations are directly generated from the original image and enable an image to be observed with different angles of view. As the multiple vector representations offer complementary information to represent the object, the simultaneous use of them is very useful. The rationale of integration of multiple vector representations can be also explained by viewing different concatenations of all rows and columns of an image as different time sequences. Moreover, the proposed weighted logarithmic sum scheme to integrate classification scores of all vector representations is very reasonable. Because of these factors, when the same dictionary learning algorithm is applied to the proposed multiple vector representations, robust and accurate classification results can be obtained. Our idea and method also provide a good research routine for other problems, demonstrating that to take attention on in-depth digging of image data is worthy of doing. In our opinion, matrix factorization algorithms for pattern representation and classification might be improved by a similar idea, because they also decompose a matrix into multiplication of two new matrices, being partially similar with dictionary learning.

Declaration of Competing Interest

None.

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