

TWO-DIMENSIONAL TECHNIQUE FOR IMAGE PRESENTATION AND ITS APPLICATION

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Abstract:

In contrast with PCA, the two-dimension presentation technique (TDP) developed recently is very efficient. With TDP, we can easily extract feature vectors of an image matrix by projecting the image matrix rather than the corresponding vector onto projecting axes. In this paper, we present complete properties of TDP in detail and the property of decorrelation associated with TDP is originally revealed. The differences and similarities between TDP and PCA are also analyzed and presented. Furthermore, Local-TDP approach is proposed to perform face recognition. Local-TDP aims to draw local characteristic of face images. Especially, Local-TDP appears to be beneficial to weaken the side effect on face recognition of varying imaging conditions. The possible reason is that the varying imaging conditions mainly bring strong difference for parts of the image, while the influence on other parts is little. As a result, the similarity between the extracted local features of two face images of one individual may become larger in comparison with holistic features of face images. The conducted experiment also indicates that Local-TDP is competent for extracting invariant features of face images with varying illumination.

Keywords:

PCA; correlation coefficient; uncorrelated features; reconstruction error

1. Introduction

In the area of pattern recognition, principal component analysis (PCA) has been of wide concern. PCA has been applied to image processing, face detection, feature extraction [1,2,3]. It was also exploited in handprint recognition, human-made object recognition and industrial robotics [4,5,6]. Besides, applications on gesture recognition, face recognition and autonomous navigation of the technique were also available [7,8,9]. PCA can transform original sample space into a new one, in which components of data are uncorrelated to each other. In practice, the PCA technique is a very powerful tool of dimensional reduction. While the technique is applied to

images, they should be expressed as vectors by concatenating rows (or columns) of the corresponding matrices in advance. Because the dimension of an image is usually large, applications on images of PCA usually bring high computational complexity. Moreover, the number of image samples is usually much less than the dimension of an image, so it is not assured that the corresponding covariance matrix of image samples can be estimated accurately [2]. For instance, if the resolution of an image is 100×100 , the dimension of the corresponding vector will be 10000. In this case, it is almost impossible to accurately estimate the covariance matrix using a small number of image samples.

To overcome the above disadvantages for PCA to be applied to images, a novel technique, TDP, has been proposed recently [2,3]. On one hand, the technique can be viewed as an extension of PCA. On the other hand, unlike the traditional PCA, this technique is devoted to directly extracting features of image matrices. That is, for feature extraction based on the developed technique, it is not needed to transform each image into a vector in advance. Instead, the covariance matrix is directly computed using the image matrices. Consequently, its eigenvectors (i.e. projecting axes) can be worked out with low computation complexity. By projecting one image matrix onto the projecting axes, we can obtain features of the image. Note that the projection of one image matrix onto a projecting axis is still a vector, called feature-vector. It is demonstrated that the new-arisen technique is computationally much more efficient than PCA. Besides, when the method is applied for feature extraction, it can correspond to good classification performance [2].

In this paper, the method that transforms an image matrix into a new two-dimensional matrix is termed two-dimension presentation (TDP) technique of images. In contrast, the traditional PCA technique may be called one-dimension presentation technique (ODP) of images, which transforms an image into a one-dimensional-vector

(i.e. the collection of all the features of one image forms a vector).

The paper is organized as follows. In section 2 the TDP technique is presented. Section 3 reveals the properties of TDP, including the decorrelation property. In section 3, we find that TDP is still formally analogical to PCA. It is helpful for us to understand that TDP is an extension of PCA. Experiments results are shown in section 4, and finally some conclusions are drawn in section 5.

2. TDP and its formulation

2.1. Introduction on TDP

As we have presented in the last section, TDP aims to transform an image matrix into a novel one. In practice, 2DPCA (two-dimensional PCA) is a successful TDP technique developed by Jian Yang, et al.[2,3]. According to [2], 2DPCA may be carried out as follows.

$$G_t = \frac{1}{M} \sum_{i=1}^M ((A_i - \bar{A})^T (A_i - \bar{A})) \quad (1)$$

is defined as the covariance matrix of image matrices.

$A_i \in R^{m \times n}$ denotes the i -th image in the collection of image samples, and \bar{A} is the mean of all the image samples. G_t is regarded as the generative matrix. In other words, the eigenvectors of G_t are taken as projecting axes. If u_1, u_2, \dots, u_d are the first d eigenvectors corresponding to the largest d eigen-values $\lambda_1 \geq \lambda_2, \dots, \geq \lambda_d$, u_1, u_2, \dots, u_d can be selected as projecting axes to extract features of image samples. Note that the projection of an image matrix A onto u_i is computed according to $v_i = Au_i$, where v_i is a vector, i.e. a feature-vector. Consequently, A can be transformed into a novel matrix $B = [v_1 \quad v_2 \quad \dots \quad v_d]$ based on all the projections of A onto the d projecting axes.

2.2. The property of 2DPCA

2DPCA is an extension of PCA, so we expect that some properties analogical to PCA can also hold for it. In the following, we will reveal new property of 2DPCA that has not been shown before. The existing properties are also presented.

Property 1. G_t defined by (1) is a non-negative definite matrix [2].

In fact, property 1 is guaranteed according to formula (1). It is noted that PCA is a technique of decorrelation. If every sample is a m -dimensional vector and PCA transforms each sample into a novel vector with the size of $n \times 1 (n \leq m)$, the n obtained components will be uncorrelated to each other. We also say that PCA can eliminate the correlation among the components of original samples. Generally, the generative matrix of PCA is defined

$$\text{to be } S_t = \frac{1}{M} \sum_{i=1}^M (X_i - \bar{X})(X_i - \bar{X})^T, \text{ where } X_i$$

denotes the i -th sample, \bar{X} is the mean of all the samples.

Actually, for 2DPCA, there is also a property very similar to the decorrelation property of PCA. We define correlation between two feature-vectors, f_i and f_j , as follows.

Definition 1. The correlation coefficient between feature-vector v_i and feature-vector v_j is defined to be

$$\rho(v_i, v_j) = \text{cov}(v_i, v_j) / (\sqrt{D(v_i) \cdot D(v_j)}),$$

where

$$\text{cov}(v_i, v_j) = E[(v_i - E(v_i))^T (v_j - E(v_j))] ,$$

$$D(v_i) = E[(v_i - E(v_i))^T (v_i - E(v_i))],$$

$$D(v_j) = E[(v_j - E(v_j))^T (v_j - E(v_j))].$$

Theorem 1. Feature-vectors are uncorrelated according to definition 1. In other words, $\rho(v_i, v_j)$ is equal to zero.

Proof.

Because $v_i = Au_i$, we have

$$\begin{aligned} \text{cov}(v_i, v_j) &= E[u_i^T [A^T - E(A^T)] [A - E(A)] u_j] \\ &= u_i^T E[[A^T - E(A^T)] [A - E(A)]] u_j \end{aligned}$$

In fact, $E[[A^T - E(A^T)] [A - E(A)]]$ is always evaluated by G_t . As a result, we have $\text{cov}(v_i, v_j) = u_i^T G_t u_j$. u_j is the j -th eigenvector of G_t , i.e. $G_t u_j = \lambda_j u_j$, so it is certain that $\text{cov}(v_i, v_j) = \lambda_j u_i^T u_j$. Obviously, if $i \neq j$, then

$\text{cov}(v_i, v_j) = 0$. As a result, $\rho(v_i, v_j) = 0$ will be assured.

Hence, 2DPCA can be considered as a technique that tries to obtain uncorrelated feature-vectors, while PCA aims at obtaining uncorrelated feature components. Moreover, according to [2], the following theorem holds.

Theorem 2. Among all potential TDP techniques, 2DPCA is optimal with the minimum reconstruction error.

On the other hand, from the point of view of mean-square error, it is also known that PCA is optimal of all potential OTP techniques. The above analysis clearly reveals the differences between 2DPCA and PCA. In addition, Theorem 1 and Theorem 2 are very helpful to understand the fact that as a TDP technique, 2DPCA is an extension of PCA.

3. Local-2DPCA approach

Face recognition is a challenging task affected by a number of factors. For example, if one of the circumstances such as lighting conditions, facial expression or pose varies, the image of the face will also vary. It makes face recognition difficult. However, these variations may mainly affect some parts of the face image, while the other parts of the same face may be almost stable. On the other hand, holistic characteristic of face images seems to vary comparably with the imaging conditions. In other words, local characteristic of face images may become more useful for face recognition in comparison with holistic characteristic [10]. Thus, it is possible for us to perform personal identity more easily using local features of face images. Hence, we develop Local-2DPCA technique for face recognition.

Simply speaking, Local-2DPCA means that the 2DPCA technique is applied for sub-images of original face images. For convenience, we divide one image into some rectangular sub-images, which are of the same size and can be presented by matrices with the same dimension.

Suppose that there are K training images in total. If each image is divided into L sub-images, we will take all the sub-images as training samples of Local-2DPCA. In other words, for the Local-2sDPCA technique, the number of the training images is KL . The mean of all the training

sub-images is $\bar{B} = \sum_{i=1}^{KL} B_i$. The covariance matrix Σ_{sub}

is defined by $\Sigma_{sub} = \frac{1}{KL} \sum_{i=1}^{KL} (B_i - \bar{B})^T (B_i - \bar{B})$.

After eigenvectors of Σ_{sub} are worked out, we can take the first C eigenvectors as projecting axes to extract feature-vectors of the training sub-images and the test sub-images. Note that for Local-2DPCA, the number of all the feature-vectors of one image will be cL , if every sub-image is projected onto all the projecting axes, respectively. If $B_{p1}, B_{p2}, \dots, B_{pL}$ are the L sub-images

of image matrix A_p , the feature-vectors of A_p can be worked out by $f_{pj} = B_{pj} u_q, q = 1, 2, \dots, c, j = 1, 2, \dots, L$. That is, the cL feature-vectors of A_p are $f_{p11}, f_{p12}, \dots, f_{p1c}, f_{p21}, f_{p22}, \dots, f_{p2c}, \dots, f_{pL1}, f_{pL2}, \dots, f_{pLc}$.

4. Experimental results

An experiment is performed for face images with pose 00 in Yale B database. For every person, there are 45 frontal face images (with pose "00"). They are sorted out from the database. Each of these images is cut as the size of 32×32 pixels. Then they are used to construct a new database, called new Yale B database. The new database is divided into 4 subsets according to the azimuth angle and the elevation angle of the light source with respect to the camera axis. The azimuth angle and the elevation angle in the images of subset 1 are both smaller than 12° . Both the azimuth angle and the elevation angle in the images of subset 2 don't exceed 25° and either of them is between 20° and 25° . Subset 3 includes the images for which the azimuth angle and the elevation angle don't exceed 50° and either of them is between 35° and 50° . Subset 4 includes the other images. Fig.1 shows images of the same face to illustrate difference between face images in different subsets. Subset 1 is regarded as the training set, and the images in the other sets are taken as test samples.



(a) Subset 1

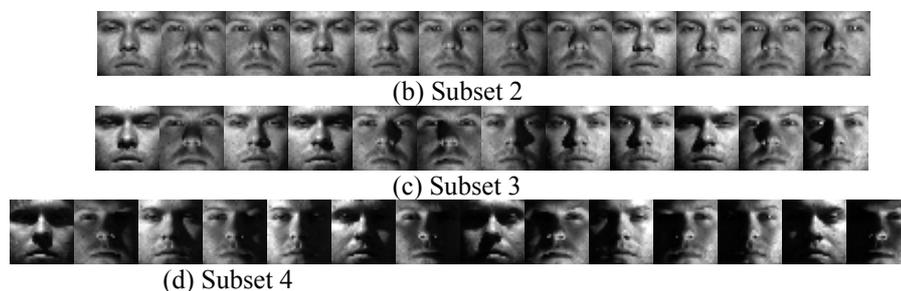
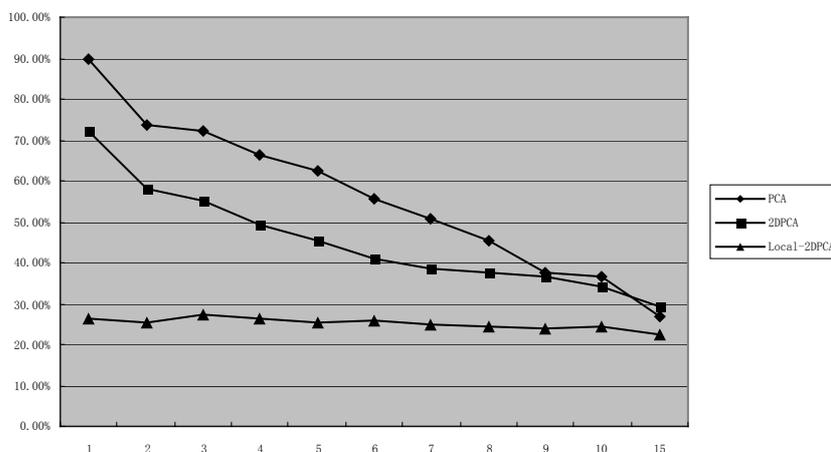


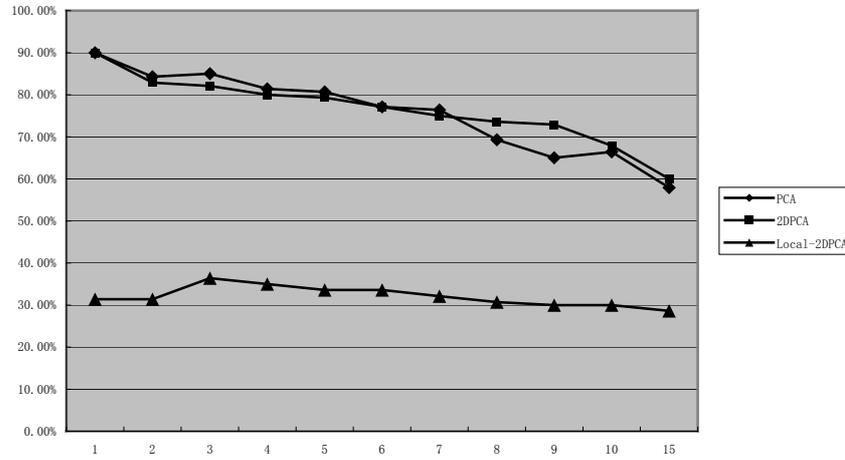
Figure 1. The face images of the same individual. (a) The images in subset 1. (b) The images in subset 2. (c) The images in subset 3. (d) The images in subset 4.

The experimental results of PCA, 2DPCA and Local-2DPCA are shown in Figure.2. Note that when Local-2DPCA is applied, each image is divided into 4 sub-images with the same size. It is revealed by Figure 2(a) that while the number of the eigenvectors used for feature extraction increases, the recognition error rates of the three methods all descend. Besides, Local-2DPCA achieves the lowest error rate among the three methods. Further, based on a few eigenvectors, the classification performance of

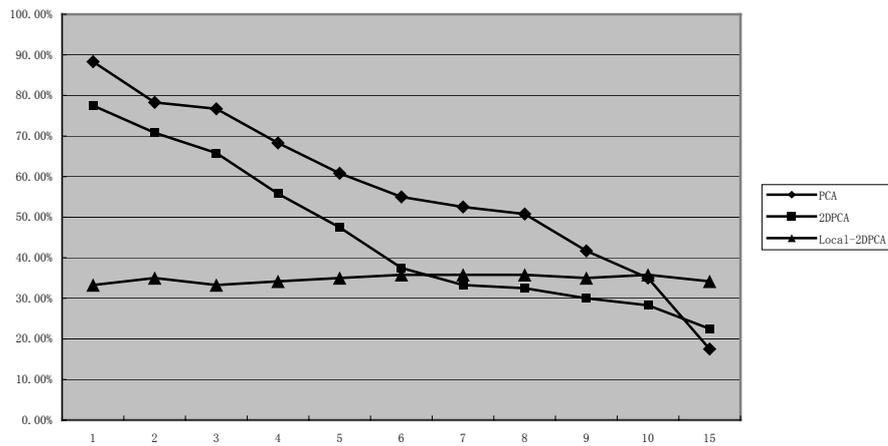
Local-2DPCA is very prominent in contrast with the other methods. Especially, (b), (c) and (d) show that the Local-2DPCA technique strongly outperforms the other two methods for the subset whose illumination is much different from the training set i.e. subset 1. It is noticeable that with other schemes of dividing image, similar results are still available. Hence, we may say that Local-2DPCA is very effective to classify the face images with varying illumination. Based on this technique, the side effect on face recognition of various illuminations can be weakened.



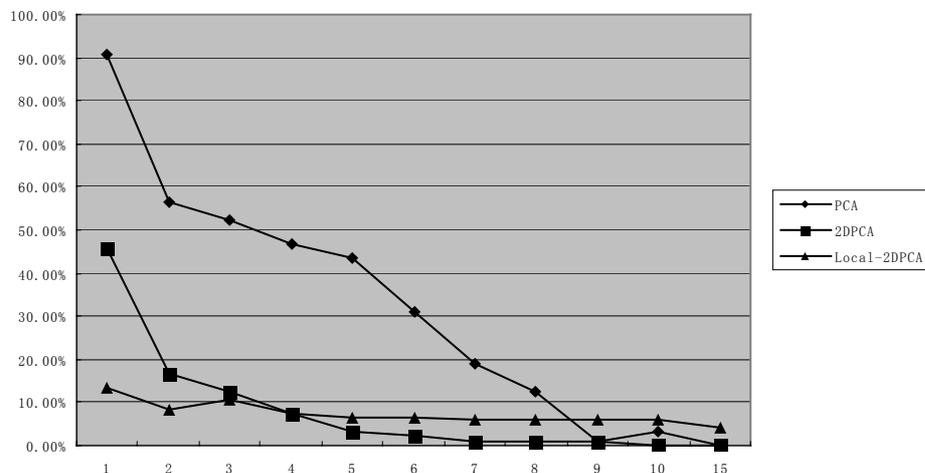
(a) The total recognition error rates of the three methods. The average of the recognition error rates on subsets 2,3,4 is called total recognition error rate.



(b) The recognition error rates on subset 4



(c) The recognition error rates on subset 3



(d) The recognition error rates on subset 2

Figure 2. Experimental results on Yale B. The x-axis denotes the number of the eigenvectors used for feature extraction, while the y-axis denotes recognition error rates.

5. Conclusion

TDP techniques can be used to directly extract features of image matrices. By using this technique, an image matrix can be transformed into a novel one with tractable computation. 2DPCA is one of the TDP techniques with successful applications. In this paper, we firstly show that the “decorrelation property” also holds for 2DPCA. That is, 2DPCA aims to obtain feature-vectors uncorrelated to each other. With this property, 2DPCA appears to be consistent with PCA in methodology.

A novel approach, called Local-2DPCA, is developed in this paper. With Local-2DPCA, we can segment a face image into some rectangular regions and perform feature extraction for all the regions, respectively. Note that for the face databases with varying illumination, Local-2DPCA can outperform 2DPCA, with higher recognition accuracy. One underlying reason is that varying illumination has different effects on different parts of a face image. Based on the extracted features, the similarity between two images of the same face can become larger, in contrast with the holistic features extracted by 2DPCA. Consequently, face recognition can be performed with higher accuracy. The experiment illustrates that our method is feasible and powerful.

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References

- [1] Kirby M., and Sirovich L., Application of the KL Procedure for the Characterization of Human Faces, *IEEE Trans. Pattern Analysis and Machine Intelligence*, Vol 12, No. 1, pp.103-108, 1990.
- [2] Jian Yang, David Zhang, A. F. Frangi, Jing-yu Yang. Two-dimensional PCA: a new approach to appearance-based face representation and recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol 24, No. 1, pp.131-137,2004.
- [3] Jian Yang, Jing-yu Yang, From image vector to matrix: a straightforward image projection technique---IMPCA vs. PCA . *Pattern Recognition*, Vol 35, No. 9, pp. 1997-1999,2002.
- [4] Murase, H., Kimura, F., Yoshimura, M., Miyake, Y., An improvement of the auto-correlation matrix in pattern matching method and its applications to handprinted ‘HIRAGANA’. *Transactions on IECE*

- J64-D(3), 1981.
- [5] Murase, H., Nayar, S., Visual learning and recognition of 3-D objects from appearance. *Int. J. Computer Vision* 14, pp. 5-24, 1995
 - [6] Weng, J. J., Crescepton and SHOSLIF: towards comprehensive visual learning. In: Nayar, S. K., Poggio, T. (Eds.), *Early visual learning* . Oxford University Press, pp. 183-214, 1996.
 - [7] J. Martin and J. Crowley, "An appearance-based approach to gesture recognition", in *Proceeding of Ninth International Conference on Image Analysis and Processing*, Florence, Italy, September, 1997, pp. 340-347.
 - [8] Yong Xu, Jing-Yu Yang, Zhong Jin, A novel method for Fisher discriminant analysis, *Pattern Recognition*, Vol 37 , No. 2, pp.381-384, 2004.
 - [9] S. Chen and J. Weng, "State-Based SHOSLIF for Indoor Visual Navigation," *IEEE Trans. Neural Networks*, Vol 11, , No. 6, pp. 1300-1314, 2000.
 - [10] Rajkiran Gottumukkal, Vijayan K. Asari. An improved face recognition technique based on modular PCA approach. *Pattern Recognition Letters*, Vol 25, pp.429-436, 2004.